TEACHER MEDIATION OF EVERYDAY MATHEMATICS IN URBAN AND RURAL KINDERGARTENS

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ABSTRACT
Early mathematics often appears in routines and play, but the pedagogical processes by which teachers make these experiences mathematically explicit remain underdescribed, particularly across contrasting resource contexts. This qualitative multi-site case study examined how teachers in one urban kindergarten in Makassar City and one rural kindergarten in Gowa Regency, South Sulawesi, integrated mathematical activities into the daily experiences of children aged 5–6 years. Five adult informants—three teachers and two principals—participated; children were observed as classroom participants, but individual learning outcomes were not measured. Data from non-participant observation, semi-structured interviews, photographs, videos, field notes, and lesson documents were analyzed through inductive first-cycle coding followed by framework-guided mapping to six Realistic Mathematics Education principles and Bishop's six mathematical activities. Across the two cases, reported practices were analytically consistent with reality, activity, levels, intertwinement, interactivity, and guidance. Counting, measuring, locating, designing, playing, and explaining were identified in routine classroom activity. The urban site relied more on structured media and teacher-directed routines, whereas the rural site drew more on natural materials, movement, and environmental exploration. These patterns suggest—without establishing causality or learning effectiveness—that different resource ecologies can support comparable forms of contextual mathematical mediation. The study contributes a non-causal analytic framework linking contextual resources, teacher mediation, and visible mathematical activity. Transferability is limited by the two-site design, incomplete child-level reporting, and the absence of measured learning gains. Future research should use larger samples, detailed video coding, multiple analysts, and longitudinal or design-based methods.

Keywords: Context-responsive pedagogy, Early childhood mathematics, Mathematical activities, Realistic mathematics education, Teacher mediation

INTRODUCTION

Background and Significance

Early mathematical knowledge is a consequential component of school readiness because it supports later reasoning and achievement across extended educational trajectories (Duncan et al., 2007; Watts et al., 2014). In early childhood, however, mathematics is not encountered only through numeral recognition or formal tasks. Children meet quantity, pattern, position, measurement, and order while distributing materials, arranging objects, moving through space, and explaining what they have done. High-quality pedagogy therefore needs to connect emerging abstractions to relationships that children can perceive and act on (Clements et al., 2023; Ginsburg, 2016).

Play-based and activity-oriented approaches provide a developmentally appropriate setting for these connections, but activity alone does not guarantee mathematical meaning. Guided play and play-responsive teaching are most productive when adults preserve children's agency while making a mathematical relationship available through materials, questions, gestures, representations, and dialogue (Fisher et al., 2013; Pyle & Danniels, 2017; Pramling et al., 2019; Weisberg et al., 2016).

This places teacher noticing at the center of early mathematics pedagogy. Teachers must identify the potential mathematical relation in an ordinary action, interpret the child's current strategy, and decide whether to prompt, model, represent, or wait. Such decisions draw on mathematical pedagogical content knowledge and on sensitivity to mathematics in play (Anders & Rossbach, 2015; Gasteiger et al., 2020; Klibanoff et al., 2006; Lee, 2010; McCray & Chen, 2012; Oppermann et al., 2016; Trawick-Smith et al., 2016).

In Indonesia, this work occurs across institutions with markedly different material and environmental resources. Urban classrooms may use commercial manipulatives, pictures, worksheets, and structured routines, whereas rural settings may draw more heavily on natural objects, movement, household practices, and community environments. Existing Indonesian studies support contextual and environment-based mathematics learning, but they also report uneven pedagogical competence and limited integration of mathematical opportunities into everyday activity (Fitria & Lestari, 2024; Iskandar et al., 2024; Kurniasari et al., 2022; Lubis & Umar, 2022; Setiawan, 2018).

Realistic Mathematics Education (RME) offers a process-oriented lens for examining how familiar situations can become sites of progressive mathematization. Its principles of reality, activity, levels, intertwinement, interactivity, and guidance emphasize meaningful contexts, children's active strategies, movement toward representation, conceptual connection, dialogue, and guided reinvention (Van den Heuvel-Panhuizen & Drijvers, 2020). In early childhood, these principles can be observed in movement from objects and actions toward relational language, pictures, patterns, and numeral symbols.

Bishop's (1988) cultural framework offers a complementary content lens by identifying counting, measuring, locating, designing, playing, and explaining as recurring mathematical activities in human practice. Recent work shows that these activities can be visible in early childhood routines, although the educational significance of an activity depends on how it is noticed and extended (MacDonald, 2026). The present study therefore examines not only which mathematical activities appeared, but also how reported teacher actions were analytically consistent with RME principles.

Research Problem and Empirical Gap

Two problems remain insufficiently resolved. First, research often evaluates designed games, manipulatives, or interventions without documenting the situated pedagogical decisions through which teachers make spontaneous activity mathematically explicit. A child may manipulate blocks, pour water, or recite number words without attending to cardinality, comparison, sequence, position, or pattern. The key empirical issue is therefore the quality and form of mediation rather than the mere presence of materials or play.

Second, institutional context is frequently treated as background rather than an analytic condition. Different resource ecologies may shape which objects, routines, and interactional opportunities are available, yet it remains unclear whether these differences alter the pedagogical structure of contextual mathematics activity. A comparison of one urban and one rural site cannot represent Indonesia as a whole; it can, however, illuminate how similar analytical principles may be expressed through different local resources.

Prior studies provide important but partial answers. Teacher-knowledge studies often rely on interviews, tests, or designed scenarios; intervention studies demonstrate outcomes under standardized conditions; and observational studies of play-based pedagogy do not always identify the mathematical content emerging across episodes (de Chambrier et al., 2021; Gasteiger et al., 2020; Pyle et al., 2017; Schmitt et al., 2018; Størksen et al., 2023; Wickstrom et al., 2019). Consequently, the relationship among contextual resources, teacher mediation, and visible mathematical activity remains underdescribed in naturalistic Indonesian early childhood settings.

Conceptual Response from Prior Research

The literature suggests four complementary responses. Guided play retains child initiative while orienting attention toward a learning goal; learning trajectories support movement from informal strategies to more explicit representations; play-responsive teaching coordinates content with meaningful participation; and teacher-child interaction can make children's emerging strategies visible (Clements & Sarama, 2007; Clements et al., 2023; Karlsson et al., 2026; Panaoura, 2026; Taner Derman et al., 2020; Van Oers, 2010; Vogt et al., 2018).

Within RME, these responses can be understood as guided reinvention. Teachers begin from situations children can experience, invite informal action and comparison, use interaction to reveal thinking, and gradually connect activity to more structured representations. This process is not equivalent to using concrete objects: it requires a pedagogical sequence that preserves children's participation while foregrounding a mathematical relation.

Bishop's categories broaden the analytic field beyond number. Counting concerns systematic quantification; measuring concerns comparison and units; locating concerns spatial relationships; designing concerns form, pattern, and construction; playing concerns rules and strategic possibilities; and explaining concerns classification, justification, and meaning-making (Bishop, 1988). These categories are used here as sensitizing descriptors rather than as an uncontested or exhaustive universal taxonomy.

Integrated Gap and Theoretical Positioning

RME and Bishop's framework answer different questions. RME helps analyze how teachers organize a contextual learning process, whereas Bishop's categories help identify what kinds of mathematical activity become visible. Used alone, RME may describe contextual pedagogy without differentiating the mathematical substance of an episode; Bishop's framework may classify an activity without explaining the pedagogical moves that made the relation explicit.

The study's novelty is therefore not the simple juxtaposition of two established frameworks. It proposes an analytic distinction among contextual resource ecology, teacher mediation, and visible mathematical activity. This distinction allows the same episode to be examined simultaneously for its material and social conditions, the teacher's pedagogical response, and the mathematical relation evidenced in children's action or language.

The resulting context–mediation–activity chain is treated as a sensitizing and recursive proposition, not a tested causal model. Context shapes available opportunities; teacher action may direct attention or reorganize participation; children's responses may in turn prompt further guidance. The framework is intended to support interpretation and future testing, while the present qualitative design can establish only context-bound patterns and plausible relationships.

Purpose, Research Questions, and Scope

The study explored how teachers integrated mathematical activities into the daily experiences of children aged 5–6 years in one urban and one rural kindergarten in South Sulawesi. It addressed two questions: (1) How were practices analytically consistent with the RME principles of reality, activity, levels, intertwinement, interactivity, and guidance? and (2) How did counting, measuring, locating, designing, playing, and explaining become visible across the two settings?

The study contributes an integrated process–content framework and a cross-case account of how different resource ecologies were associated with different forms of activity. The contribution is interpretive rather than causal: it specifies a set of relationships and teacher-noticing prompts that can be examined in subsequent research and professional learning.

The inquiry was bounded to two purposively selected institutions, five adult informants, children observed in naturally occurring classroom activity, and the documentary corpus described below. It did not measure child-level learning gains, compare program effectiveness, test statistical hypotheses, or support national generalization. Urban and rural are used as contextual descriptors for these cases rather than as population categories.

Theoretical and Analytical Framework

Contextual Mathematics, Teacher Noticing, and Assessment

Early mathematics is cognitive, social, embodied, and cultural. Relations such as amount, order, distance, shape, capacity, and repetition become mathematically significant when children attend to them, compare them, represent them, or discuss them. Contextual meaning-making is therefore not the assumption that every daily action produces learning; it is the study of how activity becomes organized around a mathematical relation (Clements et al., 2023; Ginsburg, 2016; Van Oers, 2010).

Teacher mediation involves observing a child's activity, interpreting the apparent strategy, selecting a response, and monitoring how participation changes. Mathematical pedagogical content knowledge is essential because superficially similar behavior can indicate different understandings: reciting a sequence is not the same as coordinating one-to-one correspondence, and visually comparing containers is not the same as using a unit iteratively (Gasteiger & Benz, 2018; Gasteiger et al., 2020).

Play-based pedagogy also raises an assessment problem. Teachers need evidence of children's strategies and explanations rather than relying only on task completion or worksheets. Research on play-based profiles and assessment suggests that pedagogical beliefs shape what teachers notice and document (Pyle et al., 2022). In this study, observed actions and utterances are therefore distinguished from the researchers' mathematical interpretation of those actions.

RME and Bishop as Complementary Lenses

The six RME principles provide a process-oriented vocabulary. Reality concerns experientially meaningful situations; activity concerns children's manipulation, movement, testing, and representation; levels concern progression from situated action toward models and symbols; intertwinement concerns connected mathematical domains; interactivity concerns dialogue and comparison; and guidance concerns prompts, examples, materials, and gradually reduced assistance (Van den Heuvel-Panhuizen & Drijvers, 2020).

Bishop's (1988) six activities provide a content-oriented vocabulary. They are used because the reported classroom evidence extends beyond counting to comparison, position, construction, rule-governed play, and explanation. Their use remains interpretive: the categories help organize visible activity, but they do not imply that all episodes contained equally explicit mathematics or that the taxonomy is exhaustive.

The operational distinction is as follows: RME categories describe the pedagogical process, Bishop's categories describe the mathematical form, and teacher mediation describes the situated actions through which a relation was noticed, represented, discussed, or scaffolded. An episode could therefore receive more than one mathematical classification and more than one RME code.

Integrated Analytical Proposition

The integrated proposition begins with a contextual resource ecology, including materials, routines, spatial arrangements, and interactional opportunities. Teacher mediation may then make a relation more visible through a question, example, artifact, grouping structure, or graduated assistance. Children's action and language provide the evidence from which mathematical activity is interpreted.

The relationship is recursive rather than linear. A child's response may redirect the teacher's next prompt; a material may constrain or expand the available strategy; and peer talk may alter the meaning of an activity. Figure 2 is therefore a sensitizing analytical framework for organizing evidence, not a causal pathway or validated instructional model.

This integration makes one distinction visible that either framework alone could obscure: comparable mathematical activities may arise through different local resources, while similar materials may support different learning opportunities depending on teacher mediation. The empirical task is to describe those relationships without inferring effectiveness beyond the available evidence.

METHOD

Research Design, Cases, and Unit of Analysis

This study used a qualitative multi-site case-study design focused on meaning, interaction, and situated pedagogical process (Creswell & Poth, 2018; Yin, 2018). Each institution constituted a primary case. The principal unit of analysis was the pedagogical episode in which a teacher action, child response, material or routine, and an identifiable mathematical relation could be examined together. Teacher practices and children's observable mathematical actions were treated as embedded units within each case. Cross-case comparison was analytic rather than statistical.

The urban case was Ananda Kindergarten in Makassar City, with fieldwork from September to November 2025; the rural case was Harapan Bangsa Kindergarten in Gowa Regency, with fieldwork from January to March 2026. Sites were selected purposively because they represented contrasting resource ecologies and because preliminary access indicated the presence of mathematics-related daily routines. This positive-case selection supported examination of pedagogical integration but may underrepresent settings where such mediation was absent or weak. The urban–rural labels are contextual descriptors and not independent explanatory variables.

Participants and Observational Cohorts

Five adult informants were selected purposively for direct involvement in classroom practice or institutional leadership. Ananda Kindergarten contributed two teachers and one principal; Harapan Bangsa Kindergarten contributed one teacher and one principal. Teachers were the primary sources for pedagogical episodes. Principals provided contextual and institutional corroboration and were not treated as equivalent sources of classroom-event evidence.

Children aged 5–6 years in the participating classrooms were observational participants. The analysis did not construct individual child profiles or compare achievement. The exact number and demographic composition of observed children were not documented in the materials available for this report; consequently, the study treats classroom episodes, rather than individual children or prevalence estimates, as the unit of inference. This reporting limitation materially constrains judgments about data sufficiency and transferability.

Table 1. Research sites, participants, and contextual characteristics

Dimension	Ananda Kindergarten	Harapan Bangsa Kindergarten
Location and context	Makassar City, South Sulawesi; urban case	Gowa Regency, South Sulawesi; rural case
Fieldwork period	September–November 2025	January–March 2026
Adult informants	Two teachers and one principal	One teacher and one principal
Children	Aged 5–6 years; exact count and demographics not documented	Aged 5–6 years; exact count and demographics not documented
Primary unit of analysis	Pedagogical episode nested within the institutional case	Pedagogical episode nested within the institutional case

Dimension	Ananda Kindergarten	Harapan Bangsa Kindergarten
Dominant reported resources	Structured media, manipulatives, pictures, blocks, and routines	Natural and everyday materials, movement, environmental exploration, and practical routines
Analytic purpose	Examine structured contextual mediation	Examine environmentally embedded contextual mediation

Data Collection and Evidentiary Corpus

Data were generated through non-participant observation, semi-structured interviews, and documentation. Observation focused on teacher prompts, materials, assistance, child actions, peer interaction, and moments in which quantity, size, order, position, pattern, rule, or explanation became visible. Interviews explored teachers' understandings of mathematics in routines, movement from concrete experience to symbols, use of play and environmental resources, interaction, and responses to difficulty. Documentation included photographs, video records, field notes, and planning documents such as daily lesson plans or open modules.

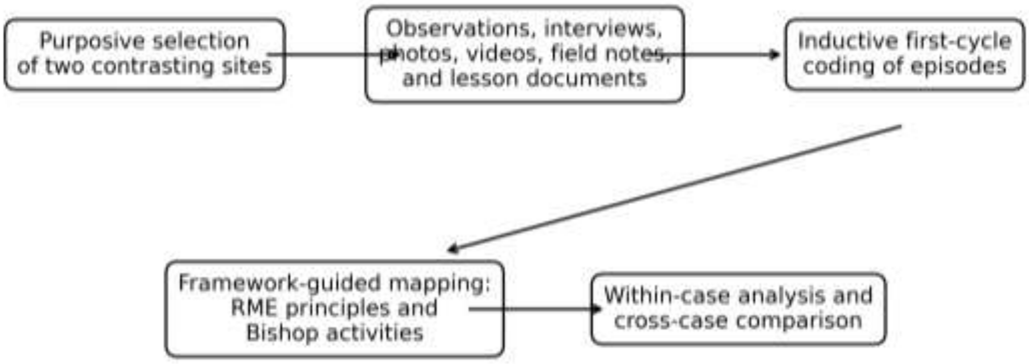

Figure 1. Qualitative multi-site case-study procedure

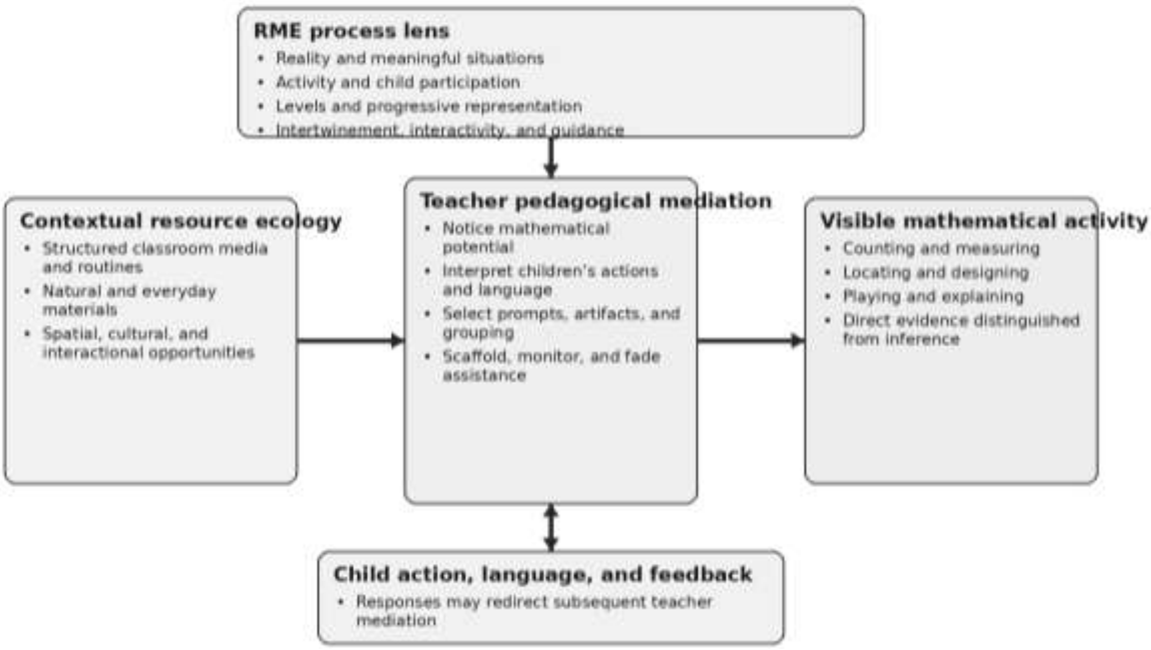

Figure 2. Sensitizing analytical framework linking contextual resource ecology, teacher mediation, and visible mathematical activity

The researcher served as the primary qualitative instrument. The available source records do not specify the number and duration of observation sessions, total hours per site, number and duration of interviews, recording and verbatim-transcription procedures, language and translation procedures, piloting of the interview guide, document-sampling criteria, or the person responsible for each data source. The different fieldwork periods may also have exposed the cases to different institutional calendars or seasonal themes. These omissions limit reproducibility and the assessment of comparable case coverage.

Data Analysis and Cross-Case Comparison

Analysis combined inductive first-cycle coding with framework-guided second-cycle coding. First, observation records, interview material, and documents were read repeatedly and coded for actions, statements, objects, interactional moves, and learning sequences. Second, related codes were grouped into categories such as real-context use, active manipulation, concrete-to-symbol progression, conceptual connection, dialogue, and scaffolding. Third, categories were mapped to the six RME principles, while a parallel pass classified visible mathematical activity through Bishop’s six categories. Finally, within-case accounts were compared to identify convergence, contextual variation, and alternative interpretations.

Frameworks were therefore introduced as sensitizing categories after initial open coding rather than presented as purely emergent themes. An episode was treated as mathematically relevant only when the available evidence indicated attention to quantity, magnitude, order, position, pattern, rule, or explanation. Overlapping classifications were permitted. Coding was conducted by the researcher; no inter-coder agreement statistic was calculated. Cross-case convergence required support from more than one evidence source and appearance in both sites, but the analysis did not use theme frequencies as measures of prevalence.

Table 2. Illustrative pathway from reported evidence to analytic classification

Reported or observed evidence	First-cycle code	Framework-guided mapping	Interpretive status
A child points to objects one by one while saying number words	Coordinates verbal sequence with objects	RME: activity/levels; Bishop: counting	Direct action plus cautious inference; not a measure of cardinality
A teacher begins with bottle caps before numeral symbols	Moves from concrete referent to representation	RME: levels/guidance; Bishop: counting	Teacher-reported sequence; no learning-gain claim
Children discuss answers in a small group after teacher prompts	Uses peer dialogue to externalize responses	RME: interactivity; Bishop: explaining	Interactional evidence; reasoning quality not systematically coded

Trustworthiness, Reflexivity, Ethics, and Data Governance

Credibility was pursued through method and source triangulation, comparison of interview accounts with observations and documents, use of photographs, video and field notes as reference materials, and reconfirmation of interview interpretations with informants. The available report does not document when member checking occurred, which materials were returned, whether disagreements arose, or which interpretations changed. Accordingly, member checking is reported as a credibility procedure but not as independently auditable evidence of consensus.

Dependability was supported by applying the same analytic distinctions across cases, retaining site-specific differences, and considering whether activity reflected mathematical attention or only physical participation. A complete codebook, reflexive diary, decision log, software record, and negative-case archive were not available. The evidence matrix and illustrative coding pathway in this report improve transparency, but they do not substitute for a fully preserved audit trail.

Because the research involved teachers and young children and included photographs and video, publication requires documented ethical authorization, informed consent, parental or guardian permission, developmentally appropriate child assent and respect for dissent, and explicit data-governance procedures. The submitted materials did not provide an ethics committee name, approval number or exemption basis, consent and assent procedures, image-use authorization, pseudonymization protocol, storage and encryption arrangements, access controls, retention period, or destruction plan. These omissions must be resolved with verified information before the manuscript is submitted.

RESULTS AND DISCUSSION

Cross-Case Context and Interpretive Boundary

Across the reported observations, interviews, and documents, both institutions used daily activity as a starting point for mathematics-related interaction. Ananda Kindergarten more often used structured media, pictures, blocks, bottle caps, loose parts, peer-counting routines, and direct questioning. Harapan Bangsa Kindergarten more often used natural or household materials, songs, movement, market play, water activity, lining up, storage routines, and environmental exploration. These descriptions identify differences in available and selected resources; they do not measure resource quality or instructional effectiveness.

The cross-case comparison suggests recurring pedagogical features amid contextual variation. In both sites, teachers began with recognizable situations, invited action on objects, used interaction to direct attention, and provided assistance when difficulty became visible. This pattern is interpreted as structural similarity in reported mediation, not as evidence that the two sites produced equivalent learning quality or outcomes.

RME-Consistent Pedagogical Practices

Reality: Beginning with Experiences Children Could Recognize

The reality principle was inferred when teachers anchored attention in objects and situations familiar to children. At Ananda Kindergarten, grains, fingers, pictures of vehicles, classmates, and travel contexts were used to make quantity or position perceptible. One teacher stated, “When counting, we usually use real objects like grains or fingers so children can see the numbers directly.” This quotation supports the use of concrete referents; it does not by itself demonstrate durable conceptual learning.

At Harapan Bangsa Kindergarten, attendance, buying-and-selling play, leaves, fish, water, oil, lining up, and storage routines supplied comparable experiential anchors. A teacher described children learning from “situations they experience directly.” The evidence indicates that realism was enacted through locally available situations rather than through a single type of manufactured material.

Across the cases, different resources appeared to serve the same analytic function: making a mathematical relation available within recognizable experience. The finding is therefore best stated as an RME-consistent pattern, because the records do not establish that teachers consciously implemented an RME program.

Activity: Learning by Manipulating, Moving, and Experimenting

Children were reported or observed counting peers, arranging objects, coloring and gluing, stacking Lego, using buttons and popsicle sticks, pouring liquids, singing, moving, and experimenting. At Ananda Kindergarten, direct questions often initiated action; at Harapan Bangsa Kindergarten, practical materials and songs coordinated speech, gesture, and manipulation.

Pointing to objects one by one while counting was interpreted as emerging coordination between number words and objects; pouring water made differences in fullness perceptible; and rearranging pictures or blocks showed attention to order and form. These interpretations are grounded in visible action, but the absence of full time-bounded transcripts limits claims about children's underlying concepts.

Levels: Progression from Concrete Action to Representation

Both sites reported movement from concrete objects toward more explicit representation. At Ananda Kindergarten, teachers used bottle caps, fingers, grains, or pictures before introducing numeral symbols. At Harapan Bangsa Kindergarten, children handled and grouped objects, counted them, and matched quantities to numerals or worksheets.

This sequence is analytically consistent with the RME principle of levels and research on the transition from informal to formal mathematical knowledge (Purpura et al., 2013). It provides evidence of a pedagogical progression from situated action toward representation, but it does not establish learning gains, retention, or equivalent progression for all observed children.

Intertwinement: Multiple Mathematical Ideas Within One Event

Several mathematical relations appeared within the same activity. Ananda Kindergarten episodes combined counting, shape or color differentiation, grouping, and order; Harapan Bangsa Kindergarten episodes combined grouping, counting, comparison, and measuring. The data therefore support conceptual intertwinement at the level of activity design and classroom interaction.

Ball-fetching, market play, and water play illustrate how movement, number, comparison, role, language, and turn-taking could co-occur. Whether children attended to every relation cannot be established from the summarized evidence, so the analysis identifies available and visible relations rather than comprehensive individual understanding.

Interactivity: Constructing Meaning Through Dialogue and Collaboration

At Ananda Kindergarten, interactivity was reported mainly through teacher-child questioning and immediate responses. At Harapan Bangsa Kindergarten, small-group work and discussion created more peer-to-peer interaction. Both forms made at least some children's actions and answers available for further teacher response.

The two interactional architectures may offer different opportunities: direct questioning can focus attention and provide rapid feedback, whereas small-group discussion can increase peer language and comparison. The study did not systematically code participation rates or discourse quality; these are analytic possibilities rather than tested advantages.

Guidance: Scaffolding Without Replacing Children's Thinking

Teachers used repetition, examples, gestures, pictures, objects, modeling, props, and reduced assistance. At Ananda Kindergarten, a teacher described returning to fingers, images, or objects when an explanation was not understood. At Harapan Bangsa Kindergarten, the reported sequence involved modeling, inviting the child to try, adding a prop if needed, and reducing assistance.

These practices were interpreted as contingent guidance because support varied in response to visible difficulty. The evidence supports the presence of adaptive assistance in selected episodes, but not its frequency, consistency across teachers, or effect on child outcomes.

Table 3. Cross-case comparison of RME-consistent pedagogical practices

RME principle	Ananda Kindergarten (urban case)	Harapan Bangsa Kindergarten (rural case)	Cautious cross-case interpretation
Reality	Concrete classroom objects, pictures, peers, and familiar transport contexts	Attendance, market play, natural objects, water, and daily routines	Different resources provided recognizable contexts; effectiveness was not tested
Activity	Counting peers, arranging, coloring, gluing, Lego, and direct response	Buttons, sticks, songs, cutting, pouring, and environmental exploration	Children participated through manipulation and movement in reported episodes
Levels	Objects before numerals; drawing before reading or writing	Concrete objects, grouping, quantity-symbol matching, and worksheets	Both cases reported concrete-to-representation sequences; child trajectories were not measured
Intertwinement	Counting, grouping, shape, and color within one task	Grouping, counting, comparing, and measuring within connected sequences	Multiple relations were available; individual attention to each relation cannot be assumed
Interactivity	Teacher-child questioning and immediate feedback	Small-group discussion and peer collaboration	Different interactional architectures made some responses visible
Guidance	Repetition with alternative media and examples	Modeling, props, guided attempts, and fading assistance	Selected episodes were consistent with contingent scaffolding; frequency and effects are unknown

Children's Mathematical Activities Interpreted Through Bishop's Framework Counting

Counting was the most frequently described mathematical activity in the report because it appeared in attendance, lining up, pictures, objects, steps, play materials, food, and cleanup routines. The urban site presented more teacher-directed counting examples, whereas the rural site offered a broader range of routine contexts.

Pointing to each object while producing number words was directly observed and interpreted as emerging one-to-one coordination. Recounting after a classmate joined made change in a set visible. Cardinality should be inferred only where children's final-count statements or related behavior were documented; the available summaries do not establish this for every episode.

Measuring

Measurement appeared through comparative language and informal units: many and few, large and small, tall and short, full, half, and empty. The reported rural activities also included hand spans, footsteps, fish size, and container capacity, while the urban examples emphasized direct comparison of nearby objects and quantities.

These episodes made magnitude and attribute comparison visible. References to unit iteration are analytic interpretations and should be restricted to episodes in which repeated use of a hand span or footstep was actually observed; no formal measurement assessment was conducted.

Locating

Locating involved position, direction, and spatial organization. Children placed classroom objects in designated locations, arranged pictures in columns, discussed places and vehicle parts, and participated in movement activities involving up–down, front–back, and right–left.

Correct placement or correction of an item's position provided direct behavioral evidence of spatial attention. The data indicate that spatial language was coordinated with action, but they do not show the stability or generalization of children's spatial concepts.

Designing

Designing was identified in arranging, constructing, sequencing, and forming wholes from parts. The urban site more often used predefined patterns, letters, Lego, and traffic-light sequences; the rural site more often used collages, cut-outs, picture parts, and open-ended natural materials.

These activities made order, repetition, form, and part–whole relationships available. The contrast suggests different opportunities for reproducing an established structure and generating a structure from available materials, but the study did not compare creativity, agency, or learning outcomes systematically.

Playing

Play functioned as a broad context in which several mathematical relations could co-occur. Ball-fetching combined numeral recognition, movement, object selection, and counting; market play involved quantity and exchange; puzzles and blocks involved shape and comparison; and experiments prompted prediction and measurement.

The mathematical value of play was inferred when objects, rules, roles, or teacher prompts directed attention to a relation. The study does not claim that every play episode was mathematically productive or that the observed forms were more effective than alternative pedagogies. Experimental evidence from number-board games also demonstrates that carefully structured play can strengthen children's numerical knowledge (Ramani & Siegler, 2008).

Explaining

Explaining appeared when children recalled an activity, answered questions, described a result, presented work, or discussed an idea with peers. The urban site relied more on short teacher-led reflection, whereas the rural site reported more small-group discussion.

These utterances provided evidence that action was becoming communicable, but the available report does not provide full discourse transcripts or a systematic analysis of reasoning quality. Explanation is therefore identified as a visible activity rather than a validated measure of conceptual understanding.

Table 4. Forms of children's mathematical activity across the two cases

Activity	Illustrative evidence	Mathematical relation made visible	Evidentiary caution
Counting	Peers, objects, pictures, steps, play materials, food, and cleanup routines	Number sequence, one-to-one coordination, and change in sets	Cardinality is inferred only when final-count evidence is present
Measuring	Comparing quantity, height, size, fullness, hand spans, and footsteps	Magnitude, attribute comparison, informal units, and capacity	No formal measurement assessment was conducted
Locating	Storage places, columns, vehicle parts, movement games, and directions	Position, orientation, and spatial relations	Stability and generalization of spatial concepts were not tested
Designing	Lego, letters, collages, color sequences, and traffic-light patterns	Pattern, order, form, and part–whole relations	Creativity and agency were not systematically compared
Playing	Ball games, market play, puzzles, blocks, and experiments	Rules, strategy, hypothetical action, and integrated relations	Not every play episode was shown to be mathematically productive
Explaining	Reflection, questions, presentation, and peer discussion	Verbal representation, classification, and emerging justification	Full discourse transcripts and reasoning-quality analysis were unavailable

Contextual Variation and Teacher Mediation

The central cross-case interpretation is that contextual variation and pedagogical mediation were analytically separable. Structured media and directed routines were more prominent in the urban case, while natural materials and environmental activity were more prominent in the rural case. In both, teachers used questions, examples, interaction, or assistance to make a mathematical relation more visible. The design supports this descriptive relationship but does not establish mediation as a causal mechanism.

This interpretation is consistent with research showing that daily activity contains mathematical potential and that teacher knowledge and interaction influence whether that potential is noticed and extended (Gasteiger & Benz, 2018; Ginsburg, 2016; MacDonald, 2026; Trawick-Smith et al., 2016). The present study adds a cross-case distinction between the resource through which an opportunity appeared and the pedagogical move through which it was elaborated.

A more cautious proposition follows: materially different settings may afford comparable forms of contextual mathematical mediation when teachers identify and extend relations that children can act on or discuss. This proposition requires testing with richer data and child-level outcomes; it should not be read as evidence that material abundance is unimportant.

Interactional Architecture, Agency, and Explanation

Ananda Kindergarten's direct teacher questioning and Harapan Bangsa Kindergarten's small-group discussion represented different interactional architectures. The former appeared to concentrate attention and permit immediate feedback; the latter appeared to create more space for peer response and collaborative language. Because participation and discourse quality were not systematically coded, these affordances remain interpretive.

The contrast is theoretically useful because it shows that interactivity is not a single technique. RME's interactivity principle can be expressed through teacher–child exchange, peer collaboration, or combinations of both. Bishop's explaining activity also depends on these structures: opportunities to justify, classify, or describe are shaped by who speaks, to whom, and in response to which prompt.

The available evidence contains little detail about unsuccessful prompts, silent children, contested answers, or episodes in which activity remained mathematically implicit. The absence of such negative-case vignettes limits theory refinement and may reflect positive-case selection or selective reporting. The study therefore avoids claiming uniform success.

Progressive Representation and Integrated Mathematical Activity

The shared movement from objects toward symbols or relational language was one of the clearest reported patterns. Bottle caps, fingers, grains, buttons, peers, pictures, water, and natural objects provided referents before children encountered numerals or more explicit comparisons. This is consistent with progressive mathematization, but the records do not establish developmental trajectories for individual children.

Intertwinement was equally important. Market play, construction, water activity, and movement linked number, comparison, position, pattern, language, and social rules. Research on play-based mathematics similarly shows that integrated activity can support mathematical engagement when a conceptual target remains visible (Fisher et al., 2013; Panaoura, 2026; Taner Derman et al., 2020; Vogt et al., 2018).

The analytic implication is not that every integrated activity should foreground all available concepts. Teachers need sufficient conceptual clarity to identify one or two focal relations, observe children's strategies, and decide whether to extend, represent, or leave an idea for later exploration.

Theoretical Contribution and Status of the Framework

The framework's main theoretical contribution is the separation and reconnection of process and content. RME identifies pedagogical principles; Bishop's framework identifies forms of mathematical activity; and the mediation construct identifies the teacher actions through which contextual activity may become more explicit. This three-part distinction supports a more precise analysis than labeling an activity merely 'contextual' or 'mathematical.'

The integrated lens generated an insight that either framework alone could obscure: similar mathematical activities were reported through different local resources, while the same resource could support different relations depending on the teacher's prompt, grouping, or representation. This is an explanatory proposition for future research rather than a confirmed causal model.

Figure 2 therefore represents a recursive sensitizing framework. Arrows denote analytic relationships and possible feedback, not temporal inevitability or causal effect. The framework was used retrospectively to organize the reported evidence and should be validated prospectively in studies with detailed episode-level coding.

Provisional Practice and Policy Design Propositions

Practical implications should be read as provisional design propositions. Professional learning may help teachers notice mathematical potential in routines, distinguish directly observed evidence from inference, and select prompts that preserve children's agency while making a relation visible. These propositions are compatible with research on guided play, play-responsive teaching, and mathematical pedagogical content knowledge (Karlsson et al., 2026; McCray & Chen, 2012; Weisberg et al., 2016).

A concrete contribution is a teacher-noticing protocol organized around five questions: What is the child doing? What mathematical relation is visible or potentially visible? What evidence supports that interpretation? What prompt, material, or peer structure could extend the relation? What response would indicate that the extension was meaningful? The protocol can be used with video cases, lesson reflection, or collaborative planning.

At institutional and policy levels, the findings justify only cautious propositions. Quality review may attend to interaction, conceptual opportunity, and use of local resources rather than treating commercial materials as a sufficient proxy for pedagogy. Decisions about resource allocation, curriculum standards, or urban–rural equity require broader samples and outcome evidence. Play-based assessment practices should document strategies and explanations in addition to worksheets (Pyle et al., 2022).

Table 5. Provisional teacher-noticing protocol derived from the integrated framework

Phase	Analytic question	Example of a cautious pedagogical response
Notice	What is the child doing or saying?	Describe the action before naming the mathematical concept
Interpret	What relation is visible or potentially visible, and what evidence supports that interpretation?	Distinguish direct evidence from inference
Extend	What prompt, artifact, representation, or peer structure could make the relation more explicit?	Ask for comparison, placement, continuation, or explanation without replacing the child's strategy
Check	What response would indicate that the extension was meaningful?	Look for coordinated action, revised strategy, relational language, or explanation
Reflect	How did context and mediation shape participation, and what alternative interpretation remains?	Record ambiguity, non-response, and negative cases for later analysis

Transferability, Alternative Interpretations, and Limitations

Transferability is reader-judged and depends on contextual similarity. The report identifies location, fieldwork period, adult informants, dominant resource patterns, and the analytic role of each site, but it lacks verified information about child numbers, classroom composition, teacher preparation, curriculum, language of instruction, and daily schedule. These omissions restrict comparison with other settings.

Case selection also creates interpretive risk. The sites were chosen partly because mathematics-related activity was already visible, which may bias the study toward successful or readily observable practice. One institution represented each contextual category, the number of teacher informants was unequal, and fieldwork occurred in different months. Urban–rural contrasts must therefore remain descriptive and case-specific.

The evidentiary corpus is insufficiently quantified. Observation hours, interview duration, transcription and translation procedures, document counts, and distribution of sources across sites were not preserved in the submitted materials. Researcher presence may have influenced practice, and the absence of full negative-case records makes it difficult to assess confirmation bias.

Evidence and Analytic Boundaries

The study did not measure child learning, teacher knowledge, frequency of themes, or comparative effectiveness. It also used a single primary analyst and did not report systematic inter-coder agreement. Triangulation and member checking support credibility, but the undocumented details of those procedures limit independent auditability.

Accordingly, claims are restricted to reported and observed patterns that were analytically consistent with RME principles and Bishop’s categories. Terms such as effectiveness, improvement, equivalence, and central mechanism are not warranted by the design.

The rural case’s use of natural materials should not be romanticized. Local resources can offer rich mathematical affordances, but resource constraints may still limit safety, accessibility, inclusion, or continuity. The study shows possibility within two cases, not sufficiency across rural education.

Limitations and Future Research

Future studies should recruit more varied institutions, including cases in which teacher mediation is weak or absent, and should report complete participant, context, and ethics information. Repeated video analysis could preserve time-bounded episodes, child language, gestures, teacher prompts, and counterexamples.

Multiple analysts, a published codebook, reflexive memos, negative-case analysis, and an evidence matrix would strengthen transparency. Longitudinal designs could document child-level trajectories, while design-based or intervention research could test whether the integrated framework improves teacher noticing, pedagogical decision-making, or child outcomes.

Cross-regional work should also examine how language, culture, disability inclusion, curriculum expectations, and material access shape the framework’s transferability. Such research is necessary before deriving broad policy conclusions.

The current evidence therefore supports a context-bound analytical contribution and a set of provisional practice prompts. It does not support statistical generalization, causal attribution, or claims of equivalent educational quality.

An alternative interpretation remains that the reported practices reflect generally responsive early childhood pedagogy rather than RME specifically. The analysis addresses this by identifying a coherent set of RME-consistent principles, but it cannot establish teachers’ intentional adoption of RME.

Bishop's categories also function as a chosen analytic lens rather than a definitive classification. Some episodes could be categorized differently, and overlapping activity was common. Future research should test category boundaries and include participants' own interpretations of what counted as mathematics.

Ethical reporting remains a publication-critical limitation. Verified approval, consent, assent, confidentiality, image-use authorization, and data-governance information must be supplied before submission.

These limitations narrow the claims but do not eliminate the value of the cross-case descriptions. Their principal value lies in identifying relationships and questions that can guide more rigorous subsequent inquiry.

CONCLUSION

This study examined how mathematics-related activity was integrated into everyday routines in one urban and one rural kindergarten in South Sulawesi. Across the available observations, interviews, and documents, teacher practices were analytically consistent with the RME principles of reality, activity, levels, intertwinement, interactivity, and guidance, while counting, measuring, locating, designing, playing, and explaining were visible through Bishop's framework. The cases differed in their dominant resources and interactional forms, yet both provided examples of contextual mathematical mediation. The study's contribution is a non-causal, recursive framework that distinguishes contextual resource ecology, teacher mediation, and visible mathematical activity, together with provisional prompts for teacher noticing and professional learning. Because the design involved two purposively selected sites, incomplete child and data-collection reporting, a single primary analyst, no measured learning outcomes, and unresolved ethics documentation, the findings should be treated as context-bound analytical propositions rather than evidence of effectiveness or national patterns. Future research should preserve episode-level video and transcript data, include multiple analysts and diverse sites, report complete ethics and participant information, and test the framework longitudinally or through design-based intervention.

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